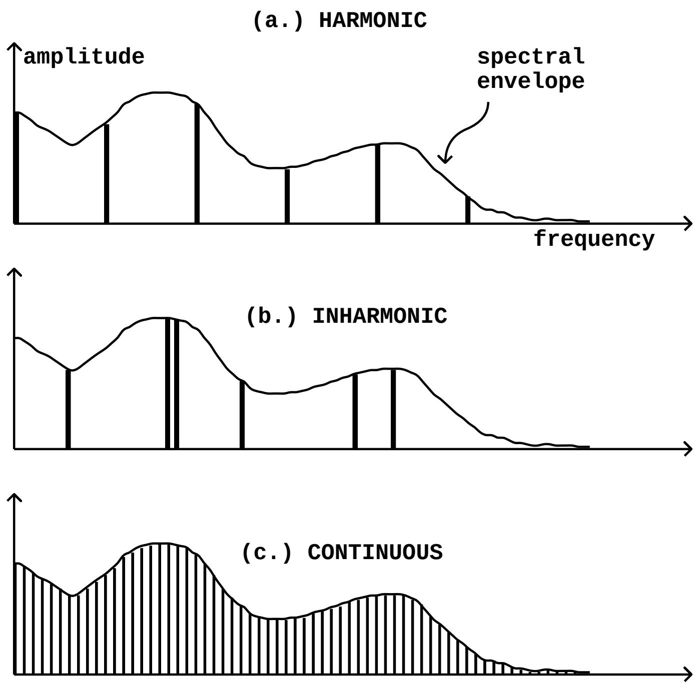
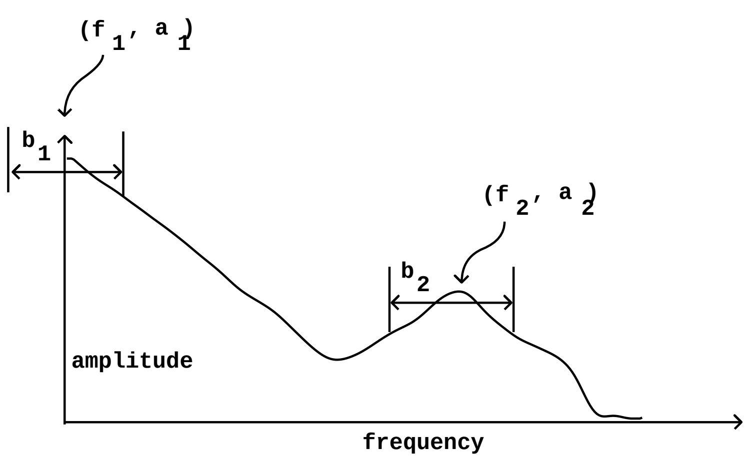
Music 170 / ICAM 103: Formula sheet #6 (no problem set yet)

Definitions:

The **spectrum** of a sound is the sound's amplitude as a "function" of frequency. A spectrum can be **discrete** or **continuous**; and if it is discrete, it can be **harmonic** or **inharmonic** as shown here:



The **spectral envelope** is a smooth curve that aproximates the spectrum (which itself might be smooth or jagged.) A peak in a spectral envelope is called a **formant**:

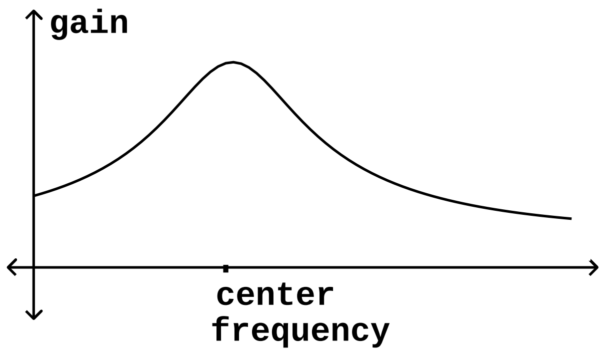


A formant has a center frequency (f_1, f_2, an amplitude (a_1, a_2, and a bandwidth (b_1, b_2.

A **Filter** is an operation on a sound that increases or decreases the amplitudes of its components, as a function of frequency. A filter thus has a frequency-dependent **gain**. The gain is the quotient,

$$h(f) = b(f)/a(f)$$

between the output amplitude at frequency f—$b(f)$—divided by the input amplitude at the same frequency, $a(f)$. The **transfer function** of the filter is its gain as a function of frequency, $h(f)$. [This is a slight oversimplification.] Here is the transfer function for one particular resonant filter:



Resonances in filters appear as peaks in the transfer function. A peak has a center frequency and a bandwidth, just like a formant (which is a peak in the spectral envelope of a sound).

The **quality** of a resonant filter (also called the **"Q"**) is the center frequency divided by bandwidth. Here the bandwidth is measured as the width of the peak once it has sloped down by 3 dB (a factor of $\sqrt{2}$ in amplitude or 2 in power).

If you filter a sinusoid, you just get another sinusoid at the same frequency. If the original amplitude is a, the new amplitude is $a \cdot h(f)$.

Masses on springs (real, damped ones) are resonant filters, whose center frequencies are just the resonant frequency of the mass/spring system, and whose Q is proportional to the number of times the spring oscillates before it damps out.