Music 170: Formula sheet and Problem Set #1 (due Oct. 2)

Definitions:

Velocity is distance travelled per unit time:

$$v = \Delta x / \Delta t$$

in units of m/sec and

acceleration is velocity change per unit time:

$$a = \Delta v / \Delta t$$

in units of m/sec^2.

Formulas:

Sinusoidal motion. Here, "x" could mean position or any other quantity. We say that x behaves as a sinusoid when:

$$x(t) = A \cdot \cos(\omega t + \phi).$$

Here A is the *amplitude*, ω the *angular frequency*, and ϕ the *initial phase*. The angular frequency is in radians per second, and the initial phase in radians. The **cycle frequency** is

$$f = \frac{\omega}{2\pi}.$$

We can re-write the expression for sinusoidal motion as:

$$x(t) = A \cdot \cos(2\pi f t + \phi).$$

The frequency f is in units of cycles per second.

Velocity and acceleration of sinusoids. If $x(t)$ is as above, the velocity is:

$$v(t) = \omega A \cos\left(2\pi f t + (\phi - \frac{\pi}{2})\right)$$

and the acceleration is:

$$a(t) = \omega^2 A \cos(2\pi f t + (\phi - \pi))$$

mass on spring. Force f is related to displacement $x = x(t)$ by Hooke's law:

$$F = -Kx,$$

and to acceleration by Newton's second law:

$$F = ma.$$

If we assume that x is a sinusoid, this gives:

$$-KA \cdot \cos(\omega t + \phi) = m\omega^2 A \cos(2\pi f t + (\phi - \pi)$$

and so:

$$\omega = \sqrt{\frac{K}{m}}.$$

Another way of looking at it is by conservation of energy. The potential energy is:

$$E_p = \frac{K}{2}x^2$$

and kinetic energy is:

$$E_k = \frac{m}{2}v^2$$

At the moment when all the energy is kinetic, the total is

$$E_k = \frac{m}{2}\omega^2 a^2$$

and when it is all potential it equals:

$$E_p = \frac{K}{2}a^2.$$

This analysis works better than the earlier one when the situation gets more complicated, e.g., when we show how the springiness of air makes sound propagate.

Problems

1. A 600 kg car goes from 0 km/h to 100 km/h in 17 seconds. What is its acceleration (in m/s^2)? What net force on the car caused this acceleration?

2. A 1.2 kg watermelon is dropped from the roof of a tall building. It takes 2 seconds to reach the ground. What is its velocity immediately before impact? What is its kinetic energy immediately before impact?

3. A spring has a spring constant of K. If the spring is cut in half, what is the spring constant of the two half-length springs?

4. A mass-spring system with two equal masses has two modes of vibration (see textbook fig. 2.7, page 27). The lowest vibrates at 20 Hz. Find the spring constant and the frequency of the second mode of vibration.

5. A partially filled bottle has a mouth with a radius of 1 cm and a neck of length 2 cm. The drinker blows over the top of the bottle and produces a frequency of 684 Hz. After drinking some of its contents, the drinker blows across the top of the bottle and produces a frequency of 484 Hz. How many liters were drunk between producing the two frequencies?