Music 170: Formula sheet and Problem Set #2 (due Oct. 9)

Definitions:

Power is energy per unit time:

$$P = E/T$$

in units of watts per second. A Watt is a Newton-Meter, or a $\mathrm{KgM}^2/\mathrm{Sec}^2$.

Intensity (of a sound) is power per unit area:

$$I = P/A$$

in units of $\mathrm{W/M}^2$ or equivalently, $\mathrm{Kg/Sec}^3$. (**NOTE:** this is not the way the word 'intensity' is used in electromagnetism—there, it means field strength, and power increases as intensity squared.)

Formulas: (see also homework 1 for sinusoidal motion)

mass on spring. Here is the derivation of the frequency of a mass vibrating on a spring. Force f is related to displacement $x = x(t)$ by Hooke's law:

$$F = -Kx,$$

and to acceleration by Newton's second law:

$$F = ma.$$

If we assume that x is a sinusoid, this gives:

$$-KA \cdot \cos(\omega t + \phi) = m\omega^2 A \cos(2\pi f t + (\phi - \pi)$$

and so:

$$\omega = \sqrt{\frac{K}{m}}.$$

Another way of looking at it is by conservation of energy. The potential energy is:

$$E_p = \frac{K}{2} x^2$$

and kinetic energy is:

$$E_k = \frac{m}{2} v^2$$

At the moment when all the energy is kinetic, the total is

$$E_k = \frac{m}{2} \omega^2 a^2$$

and when it is all potential it equals:

$$E_p = \frac{K}{2} a^2.$$

This analysis works better than the earlier one when the situation gets more complicated, e.g., when we show how the springiness of air makes sound propagate.

Wavelength and frequency.

The wavelength of a sound (written λ) depends on its frequency:

$$\lambda = v/f$$

where v is the sound's velocity and f is its frequency in cycles per second.

The period of anything that repeats (such as a sinusoid) is the time for it to make one entire cycle:

$$\tau = 1/f$$

Trigonometric identities.

Here are two useful formulas showing what happens when you add two cosines:

$$\cos(a) + \cos(b) = 2\cos(\frac{a+b}{2})\cos(\frac{a-b}{2})$$

and what happens when you multiply them:

$$\cos(a)\cos(b) = \frac{1}{2}\left[\cos(a+b) + \cos(a-b)\right]$$

Problems

1. Two sinusoids have periods of 1 msec and 1.5 msec, respectively. What is the period of the sum of the two?
2. Two equal-amplitude, equal-frequency sinusoids add up to a third whose amplitude is no different from the original sinusoids. What is the phase difference between the two original sinusoids?
3. How long would you cut a pipe, open at both ends, so that it sounds (i.e., produces a fundamental) at 440 Hz.? What would the lowest-frequency component be if you then stopped one end?
4. Human hearing theoretically works on sinusoids with frequencies ranging from 20 to 20,000 cycles per second. What wavelengths do these limits correspond to?
5. Suppose you multiplied a unit-amplitude, 100 Hz sinusoid with a unit-amplitude, 300 Hz sinusoid. What are the frequencies and amplitudes of the sinusoidal components of the product (the result of the multiplication)?