

Music 170: Formula sheet and Problem Set #3 (due Oct. 16)

Definitions:

An **interval** is a ratio between two frequencies. The interval between two musical tones is the ratio between their fundamental frequencies.

A *perfect* interval is the ratio between two small integers (typically below 6). Commonly used perfect intervals are:

name	ratio
unison	1:1
octave	2:1
minor third	6:5
major third	5:4
fourth	4:3
fifth	3:2

A **semitone** (or “half-step”) is $1/12$ of an octave:

$$h = 2^{1/12} = \sqrt[12]{2} \approx 1.05946$$

A **cent** is one hundredth of a semitone:

$$2^{1/1200} = \sqrt[1200]{2} \approx 1.000578$$

The **Western tempered scale** is a sequence of pitches all separated by semitones. The nomenclature is as shown:

pitch	semitones
A	0
B	2
C	3
D	5
E	7
F	8
G	10
A (octave)	12

So, for instance, the interval from C to the next higher E is 4 semitones.

The reference pitch is an A tuned to 440 cycles per second; this determines all the other pitches in the scale. For instance, the C below A 440 is 9 semitones down, so its frequency is:

$$\frac{440}{h^9}$$

where h is the semitone ratio shown above.

Problems

1. Two periodic tones (with fundamental and overtones present) are played at 300 Hz and 430 Hz respectively. Where are the first two partials within a minor third of each other? Within 50 Hz?
2. Assume you can hear beating between two sinusoids once the rate of beating falls below 20 Hz. What is the highest pitch at which a half-tone interval beats audibly?
3. A perfect fifth has a ratio of 3:2. A perfect major third has a ratio of 5:4. If we used an equal tempered scale with 17 notes per octave, how far from perfect will its 3rds and 5ths be? (I.e., how far off, in cents, is the best approximation you can make of the perfect interval using notes in the scale?) What do the answers become if we use a 25 note scale instead of 17?
4. What pitch from the standard Western tempered scale is closest to 1000 Hz? Is it higher or lower? By how many cents?
5. Two complex, harmonic tones, middle C and the G above it, are sounded simultaneously. Which are the lowest harmonics of the two that would beat, and how fast? (G is 7 semitones above C).