Music 170 / ICAM 103: Formula sheet and Problem Set #4 (due Oct. 23)

Definitions:

Power is energy per unit time, measured in units of **watts**, which are Joules per second. (See Rossing chapter 5.)

The **intensity** of a sound (a plane wave, for example) is the power per unit area, in units of watts per meter squared.

For a sinusoidal signal:

$$x(t) = a\cos(2\pi f t + \phi)$$

the power is proportional to the amplitude squared:

$$W = ka^2$$

where k is a constant of proportionality that depends on the physical nature of the signal.

If W_0 is a reference power and W is the power of a signal or sound, then the **power level** in decibels (dB) is:

$$L = 10\log_{10}(W/W_0)$$

If several sinusoids are added into a single signal—and if the frequencies are all different—the power of the sum is the sum of the powers. For instance, if the amplitudes of two sinusoids are a_1 and a_2 the power of the sum is

$$W = k\left({a_1}^2 + {a_2}^2\right)$$

Here is the logarithm formula:

$$\log_a(b) = \frac{\log_c(b)}{\log_c(a)}$$

The following are equivalent:

$$n = \log_h(R) = \frac{\log(R)}{\log(h)}$$

where R is an interval as a ratio, n is the interval in half steps, and $h = \sqrt[12]{2}$, and the Rossing version (P. 182):

$$I = \frac{1200}{\log(2)}\log(R)$$

where I is the interval in cents: $I = 100n$.

Problems

1. A complex periodic tone (i.e., one that potentially contains all harmonics) with a fundamental frequency of 300 Hz is multiplied by a 200 Hz sinusoid. Is the result periodic and, if so, what is its fundamental frequency? Can all of its harmonics have non-zero amplitude or, if not, which ones must be missing?

2. A synthesizer is playing sinusoids of frequencies 350, 440, 660, 700, 750, 880, 1050, and 1100. Which fundamental frequencies will a listener hear?

3. Suppose a stereo system at gain setting "11" puts out twice the amplitude it puts out at gain setting "10." How does the power compare? How many decibels more will come out?

4. Supposing that the just noticeable difference in pitch is 1/8 semitone, and that hearing ranges from 20 to 20,000 cycles per second, how many JNDs fit into the range of human hearing? Assuming the JND for loudness is 2 decibels, and that reasonable sounds vary from 0 to 100 dB in power, how does the pitch resolution of the ear compare with the resolution with which we hear loudness?

5. A musical tone contains four partials, each at 50 decibels power level. What is the power level of the tone?