Music 170: Formulas for week 1

Definitions:

Velocity is distance travelled per unit time:

$$v = \Delta x / \Delta t$$

in units of m/sec and

acceleration is velocity change per unit time:

$$a = \Delta v / \Delta t$$

in units of m/sec^2.

Force is acceleration times mass (in Newtons, i.e., Kg. meter per second squared)

Pressure is force per unit area (in Newtons per square meter)

$$P = F/A$$

Density is mass per unit volume (Kg per cubuic meter)

$$\rho = m/V$$

Formulas:

Sinusoidal motion. Here, "x" could mean position or any other quantity. We say that x behaves as a sinusoid when:

$$x(t) = A \cdot \cos(\omega t + \phi) .$$

Here A is the *amplitude*, ω the *angular frequency*, and ϕ the *initial phase*. The angular frequency is in radians per second, and the initial phase in radians. The **cycle frequency** is

$$f = \frac{\omega}{2\pi}.$$

We can re-write the expression for sinusoidal motion as:

$$x(t) = A \cdot \cos(2\pi f t + \phi) .$$

The frequency f is in units of cycles per second.

Velocity and acceleration of sinusoids. If $x(t)$ is as above, the velocity is:

$$v(t) = \omega A \cos\left(2\pi f t + (\phi - \frac{\pi}{2})\right)$$

and the acceleration is:

$$a(t) = \omega^2 A \cos(2\pi f t + (\phi - \pi))$$

mass on spring. Force f is related to displacement $x = x(t)$ by Hooke's law:

$$F = -Kx,$$

and to acceleration by Newton's second law:

$$F = ma.$$

If we assume that x is a sinusoid, this gives:

$$-KA \cdot \cos(\omega t + \phi) = m\omega^2 A \cos(2\pi f t + (\phi - \pi))$$

and so:

$$\omega = \sqrt{\frac{K}{m}}.$$

The vibration frequency, in cycles per second, is:

$$f = \frac{1}{2\pi}\sqrt{K/m}$$

Another way of looking at it is by conservation of energy. The potential energy is:

$$E_p = \frac{K}{2}x^2$$

and kinetic energy is:

$$E_k = \frac{m}{2}v^2$$

At the moment when all the energy is kinetic, the total is

$$E_k = \frac{m}{2}\omega^2 a^2$$

and when it is all potential it equals:

$$E_p = \frac{K}{2}a^2.$$

This analysis works better than the earlier one when the situation gets more complicated, e.g., when we show how the springiness of air makes sound propagate.

Hemholtz resonator. Mass is neck volume times air density

$$m = la\rho,$$

Force is change in pressure times area. If the air moves down d units:

$$\Delta F = \left[\frac{V + ad}{V}P - P\right] a$$

$$= \frac{a^2 P d}{V}$$

and the spring constant K is Δf divided by d:

$$K = \frac{a^2 P}{V}$$

Frequency is

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}} = \frac{1}{2\pi}\sqrt{\frac{P}{\rho}}\sqrt{\frac{a}{Vl}}$$
$$= \frac{v}{2\pi}\sqrt{\frac{a}{Vl}}$$

Here is a sample calculation: a bottle has a neck 1.7 cm across (so the area is $a = 2.27\text{cm}^2$ and a length of $l = 6cm$, and and holds a volume $v = 150cc$. Then we get:

$$\sqrt{\frac{a}{Vl}} = 0.05/\text{cm} = 5/\text{meter}$$

so, if the speed of sound is 343 M/sec, we get:

$$f = 343 \cdot 5/(2 * 3.14) = 273/\text{sec}$$