Music 170: Formulas for week 2

Definitions:

Power is energy per unit time:

$$P = E/T$$

in units of watts per second. A Watt is a Newton-Meter, or a $\mathrm{KgM}^2/\mathrm{Sec}^2$.

Intensity (of a sound) is power per unit area:

$$I = P/A$$

in units of $\mathrm{W/M}^2$ or equivalently, $\mathrm{Kg/Sec}^3$. (**NOTE:** this is not the way the word 'intensity' is used in electromagnetism—there, it means field strength, and power increases as intensity squared.)

Formulas:

Mass on spring. Here is the derivation of the frequency of a mass vibrating on a spring. Force f is related to displacement $x = x(t)$ by Hooke's law:

$$F = -Kx,$$

and to acceleration by Newton's second law:

$$F = ma.$$

If we assume that x is a sinusoid, this gives:

$$-KA \cdot \cos(\omega t + \phi) = m\omega^2 A \cos(2\pi f t + (\phi - \pi)$$

and so:

$$\omega = \sqrt{\frac{K}{m}}$$

Mass on spring using energy. Another way of looking at it is by conservation of energy. The potential energy is:

$$E_p = \frac{K}{2}x^2$$

and kinetic energy is:

$$E_k = \frac{m}{2}v^2$$

At the moment when all the energy is kinetic, the total is

$$E_k = \frac{m}{2}\omega^2 a^2$$

and when it is all potential it equals:

$$E_p = \frac{K}{2}a^2$$

Equating the two gives:

$$\omega = \sqrt{\frac{K}{m}}$$

This analysis works better than the earlier one when the situation gets more complicated, e.g., when we show how the springiness of air makes sound propagate.

Wavelength and frequency.

The wavelength of a sound (written λ) depends on its frequency:

$$\lambda = v/f$$

where v is the sound's velocity and f is its frequency in cycles per second.

The period of anything that repeats (such as a sinusoid) is the time for it to make one entire cycle:

$$\tau = 1/f$$

Trigonometric identities.

Here are two useful formulas showing what happens when you add two cosines:

$$\cos(a) + \cos(b) = 2\cos(\frac{a+b}{2})\cos(\frac{a-b}{2})$$

and what happens when you multiply them:

$$\cos(a)\cos(b) = \frac{1}{2}\left[\cos(a+b) + \cos(a-b)\right]$$