

## Music 170: Formulas for week 6

Recall that a **semitone** (or “half-step”) is  $1/12$  of an octave:

$$h = 2^{1/12} = \sqrt[12]{2} \approx 1.05946$$

and a **cent** is one hundredth of a semitone:

$$2^{1/1200} = \sqrt[1200]{2} \approx 1.000578$$

Suppose for example you have an interval such as a perfect minor third, which is the ratio 6:5. The number of semitones is then:

$$\log_h(6/5)$$

i.e., the power you must raise the half-step  $h$  to to reach the ratio 6:5. Using the master logarithm formula you get:

$$\log_h(6/5) = \log(6)/\log(h) - \log(5)/\log(h).$$

Here, “log”, without a specified base, could mean “log to the base 10” or “log to any other base you wish”.

Since  $h = \sqrt[12]{2}$ , or in other words,  $h^{12} = 2$ , we get:

$$\log(h) = \frac{\log(2)}{12}$$

Continuing with the example of the perfect minor third, the first term  $\log(6)/\log(h)$  comes to 31.0195 semitones (about two octaves, 24 semitones, plus seven more) minus 27.8631 semitones (about two octaves and four semitones), which comes to 3.1562 semitones.