Music 170: Formulas for week 3

Power and intensity of sinusoidal signals.

For a sinusoidal signal:

$$x(t) = a\cos(2\pi f t + \phi)$$

the power is proportional to the amplitude squared:

$$W = ka^2$$

where k is a constant of proportionality that depends on the physical nature of the signal. Intensity (power per unit area) varies the same way.

If W_0 is a reference power and W is the power of a signal or sound, then the **power level** in decibels (dB) is:

$$L = 10\log_{10}(W/W_0)$$

If several sinusoids are added into a single signal—and if the frequencies are all different—the power of the sum is the sum of the powers. For instance, if the amplitudes of two sinusoids are a_1 and a_2 the power of the sum is

$$W = k\left({a_1}^2 + {a_2}^2\right)$$

If two sinusoids have the SAME frequency, the power of the sum depends on the relative phase of the two. Suppose their phase differs by ϕ radians. Without loss of generality we can write the sum as:

$$x(t) = a\left[\cos(2\pi f t + \phi/2) + \cos(2\pi f t - \phi/2)\right]$$

which, by the first identity for cosines (from the notes for last week), comes to:

$$x(t) = 2a\cos(\phi/2)\cos(2\pi f t)$$

This shows that the amplitude of the sum is the original amplitude, times a factor $2\cos(\phi/2)$. The power is thus multiplied by $4\cos^2(\phi/2)$.

Beating.

Now suppose we add two sinusoids with close, but unequal, frequencies. Suppose they differ by g Hz, so we can write them as $f + g/2$ and $f - g/2$. Ignoring the initial phase difference for simplicty, we get

$$x(t) = a\left[\cos(2\pi(f + g/2)t) + \cos(2\pi(f - g/2)t)\right]$$

which, by the same formula becomes:

$$x(t) = 2a\cos(2\pi(g/2)t)cos(2\pi f t)$$

This can be thought of as a sinusoid of frequency f, whose amplitude varies in time. The amplitude reaches its maximum in absolute value ($2a$ or $-2a$) twice per cycle, so we hear a relative maximum in amplitude g times per second.