Music 170: notes for week 6

If a collection of sinusoids are added together, the individual sinusoids are called *partials* or *components*.

If they add up to a periodic tone, the *fundamental* is the first component, whose frequency is that of the combined tone. Some people use the word *overtone* to mean a component other than the fundamental.

An **interval** is a ratio between two frequencies. The interval between two musical tones is the ratio between their fundamental frequencies.

A *perfect* interval is the ratio between two small integers (typically below 6). Commonly used perfect intervals are:

name	ratio
unison	1:1
octave	2:1
minor third	6:5
major third	5:4
fourth	4:3
fifth	3:2

A **semitone** (or "half-step") is 1/12 of an octave:

$$h = 2^{1/12} = \sqrt[12]{2} \approx 1.05946$$

A **cent** is one hundredth of a semitone:

$$2^{1/1200} = \sqrt[1200]{2} \approx 1.000578$$

The **Western tempered scale** is a sequence of pitches all separated by semitones. The nomenclature is as shown:

pitch	semitones
A	0
B	2
C	3
D	5
E	7
F	8
G	10
A (octave)	12

So, for instance, the interval from C to the next higher E is 4 semitones.

The reference pitch is an A tuned to 440 cycles per second; this determines all the other pitches in the scale. For instance, the C below A 440 is 9 semitones down, so its frequency is:

$$\frac{440}{h^9}$$

where h is the semitone ratio shown above.

Here is how to change bases of logarithms:

$$\log_a(b) = \frac{\log_c(b)}{\log_c(a)}$$

The following are equivalent:

$$n = \log_h(R) = \frac{\log(R)}{\log(h)}$$

where R is an interval as a ratio, n is the interval in half steps, and $h = \sqrt[12]{2}$, and the Rossing version (P. 182):

$$I = \frac{1200}{\log(2)}\log(R)$$

where I is the interval in cents: $I = 100n$.

Recall that a **semitone** (or "half-step") is 1/12 of an octave:

$$h = 2^{1/12} = \sqrt[12]{2} \approx 1.05946$$

and a **cent** is one hundredth of a semitone:

$$2^{1/1200} = \sqrt[1200]{2} \approx 1.000578$$

Suppose for example you have an interval such as a perfect minor third, which is the ratio 6:5. The number of semitones is then:

$$\log_h(6/5)$$

i.e., the power you must raise the half-step h to to reach the ratio 6:5. Using the master logarithm formula you get:

$$\log_h(6/5) = \log(6)/\log(h) - \log(5)/\log(h).$$

Here, "log", without a specified base, could mean "log to the base 10" or "log to any other base you wish".

Since $h = \sqrt[12]{2}$, or in other words, $h^{12} = 2$, we get:

$$\log(h) = \frac{\log(2)}{12}$$

Continuing with the example of the perfect minor third, the first term $\log(6)/\log(h)$ comes to 31.0195 semitones (about two octaves, 24 semitones, plus seven more) minus 27.8631 semitones (about two octaves and four semitones), which comes to 3.1562 semitones.