Modulation: FM Synthesis

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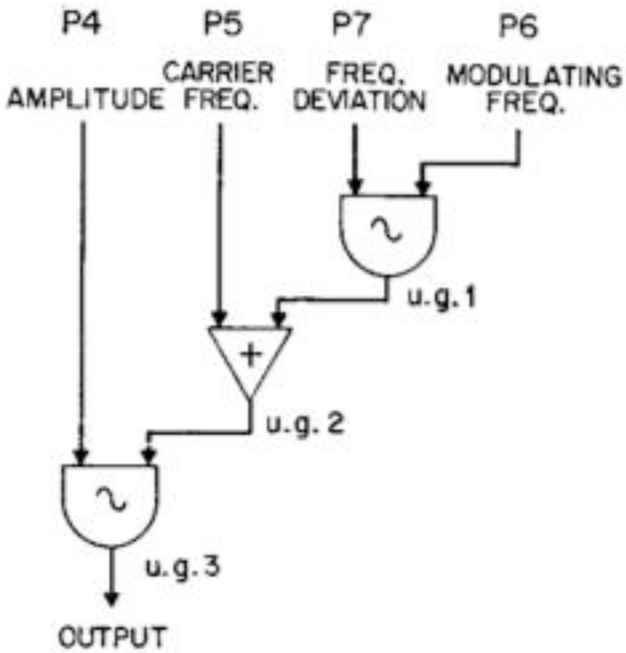
1. Overview

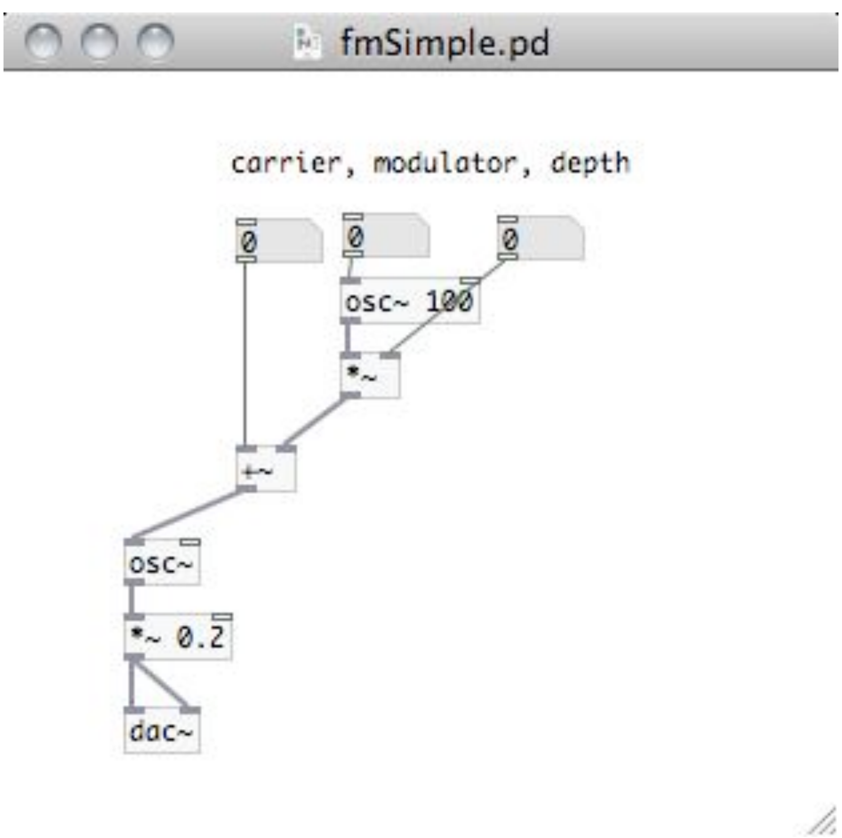
- Foundations
- Analog FM
- Digital FM / Chowning
- Extensions

2. Frequncy Modulation

- Modulate the frequency of an oscillator
- A surprising array of sounds
- FM, like AM, has a distinctive and recognizable sound
- FM, unlike AM, produces many sidebands, and the amplitude of the carrier and the sidebands changes with the depth of modulation
- Not one technique, but many
- Most simple implementation

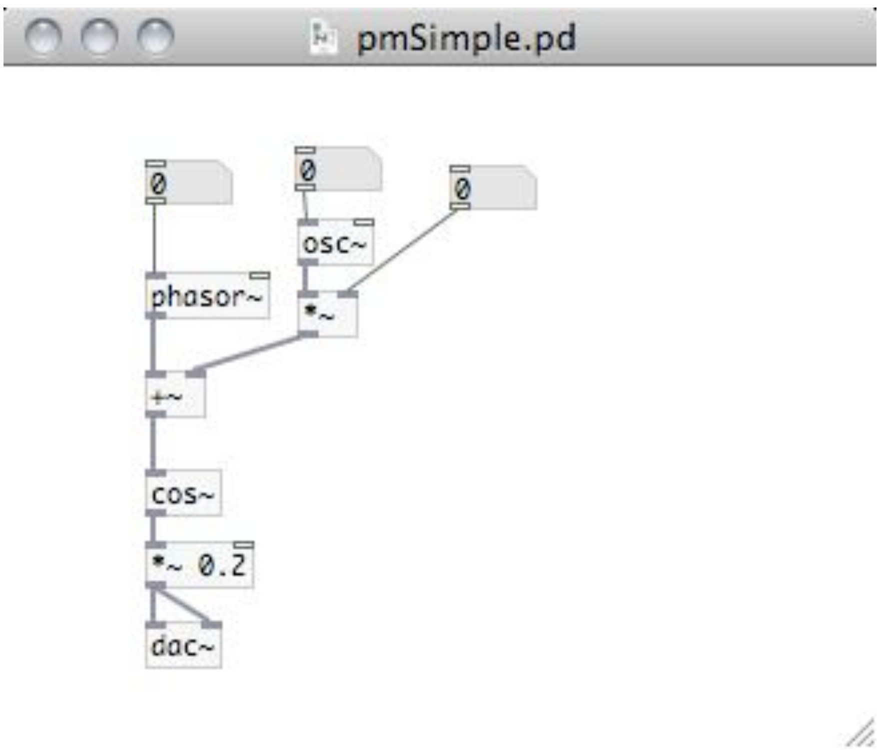


- Example: fmSimple.pd



3. Phase Modulation

- Both FM and PM are kinds of angle modulation (Loy 2007)
- "In sound synthesis, where we can do what we want with the modulating signal, there is no essential difference between frequency and phase modulation ... there is no difference in run-time computational expense or accuracy" (Schottstaedt n.d.)
- Example: pmSimple.pd



4. Lessons from FM

- The importance of parameter reduction and organization
- A source of models of modulation applicable to any parameter
- Not a finished project
- An efficient source of rich tones for usage in other applications

5. Modulation

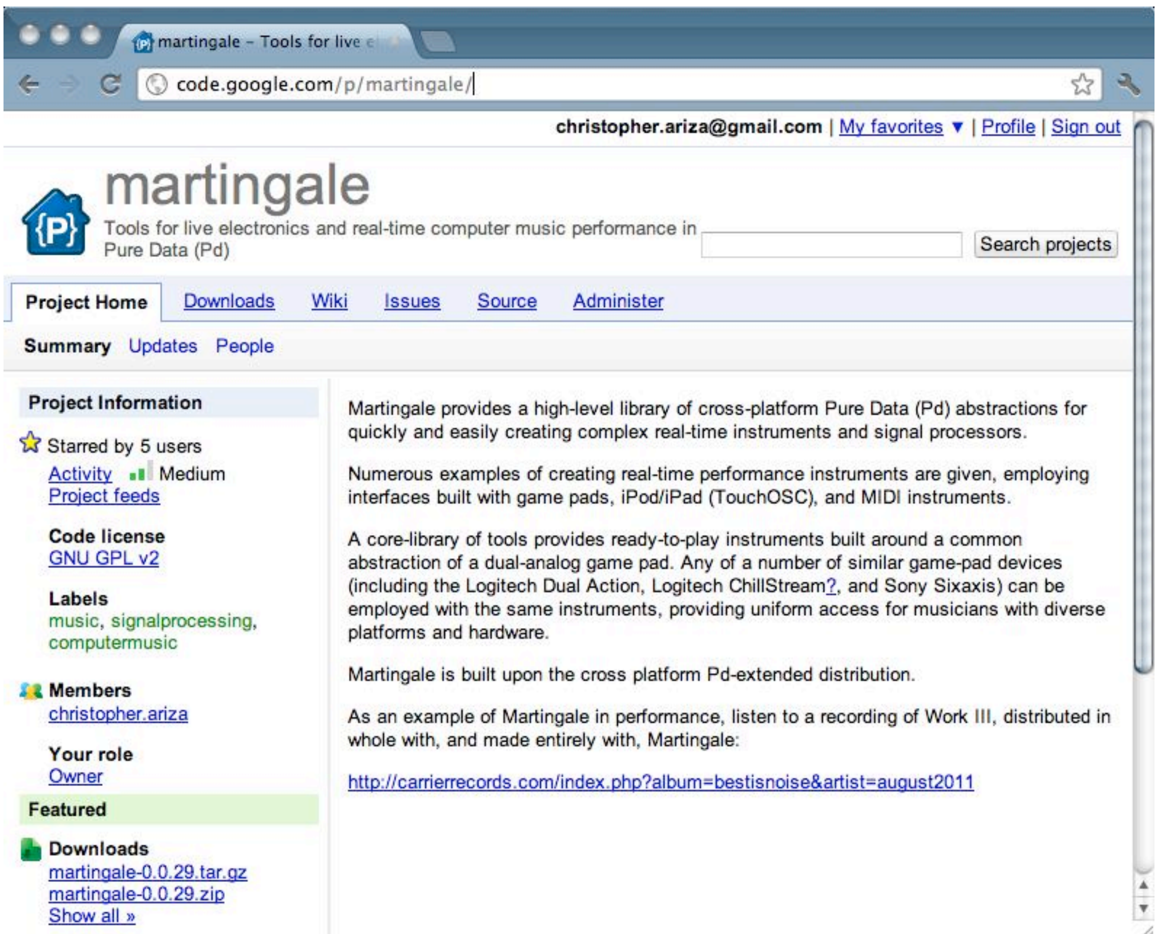
- Why do we modulate?

6. Preliminaries

- A few notes on my Pd patches

7. Preliminaries: Martingale

- An open-source library of Pd abstraction for use with Pd-extended
- All abstractions given a name-space with “mg” prefix
- Utilities for using the dual analog as a controller
- Available at Google code



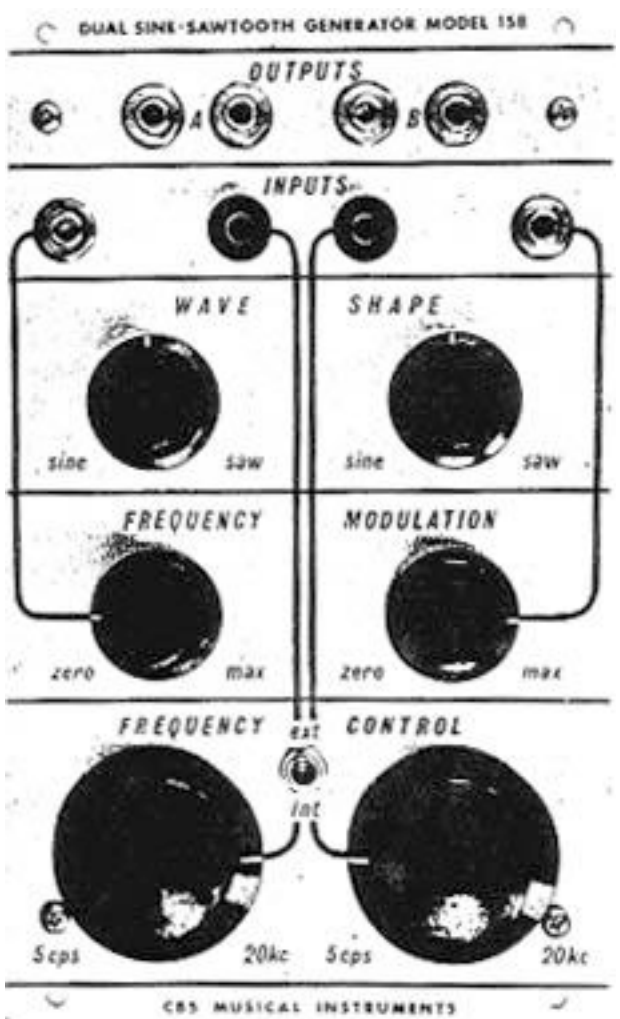
8. Preliminaries: Smoothing Signals

- Use of [lop~ 30.] to smooth discontinuities in floating point event data
- More analog alternative to line~

- Example: smoothing.pd

9. Analog Frequency Modulation

- Early modular synths in the 1960s had voltage-controlled oscillators
- These performed for both sub-audio and audio-rate modulation
- Buchla 158 Dual Sine-Sawtooth Oscillator, part of 100-series Electronic Music Box (1963)



- Voltage control was designed to map linear control voltages to exponential changes in frequency
- Sub-audio rate FM used for simulating vibrato and creating trills (Strange 1983)

10. Analog Vibrato

- Given a VCO, vary the frequency by adding and subtracting an exponential voltage from a constant (DC) voltage
- Creates an equal-sized pitched shift up and down
- In Pd, must do modulation with pitch values, the convert to frequency [mtof~]
- Example: fmAnalogVibrato.pd

11. Analog Vibrato with Envelopes

- More common usage was to modulate the vibrato with an amplitude envelope
- Creating dynamic change of sound over duration of note event
- Example: fmAnalogVibratoEnvl.pd

12. Exponential FM

- Sidebands results from exponential audio rate modulation FM
- Spacing of sidebands determined by difference in the carrier and modulator
- One octave modulation, for c = 400 and m = 1 octave, results in sidebands at 200 and 800
- Exponential FM results in non-symmetrical sidebands
- Example: fmExponential.pd

13. Linear FM

- With the use of digital oscillators on computers, linear FM became easy
- Frequency change above and below carrier is equal: c = 440 and m = 220 (Strange 1983)

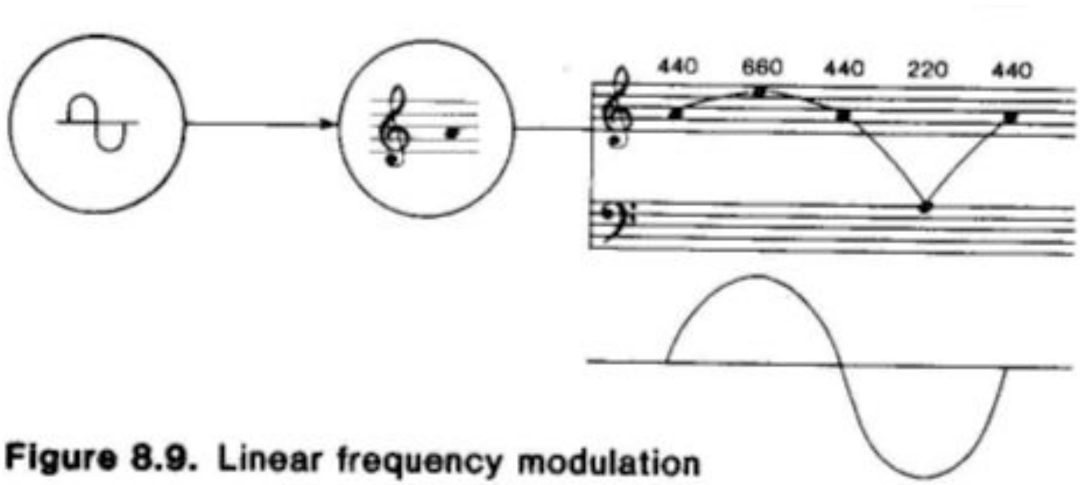
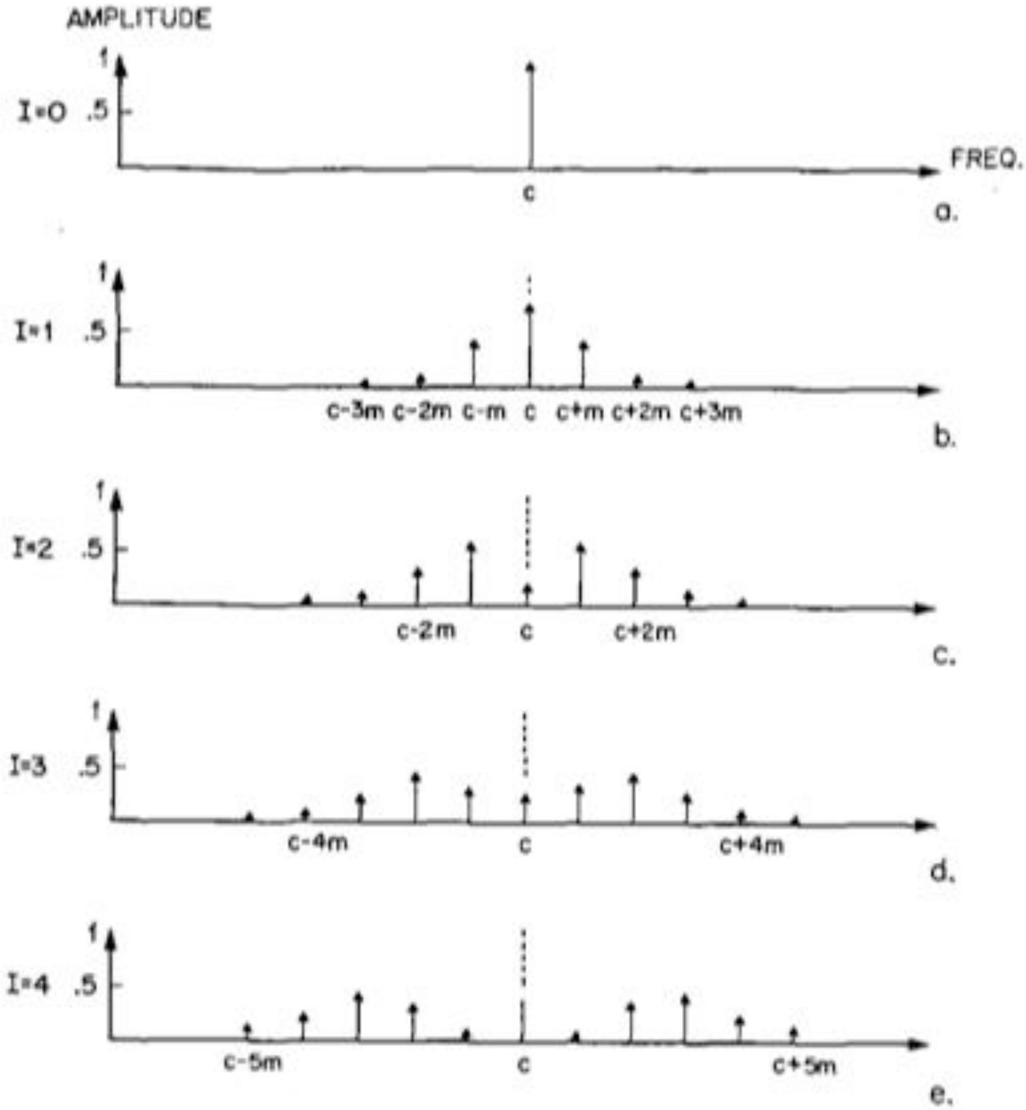


Figure 8.9. Linear frequency modulation

- Special analog oscillators with linear voltage control were later developed

14. Linear FM: Sidebands

- As modulation increases, sidebands get louder, bandwidth increases (Chowning 1973)



- Sidebands are evenly spaced multiples of modulator above and below the carrier

  Example: $c = 800$, $m = 200$

$C = 800$ Hz	(carrier)
$C + M = 1000$ Hz	(sum)
$C + (2 \times M) = 1200$ Hz	(sum)
$C + (3 \times M) = 1400$ Hz, etc.	(sum)
$C - M = 600$ Hz	(difference)
$C - (2 \times M) = 400$ Hz	(difference)
$C - (3 \times M) = 200$ Hz, etc.	(difference)

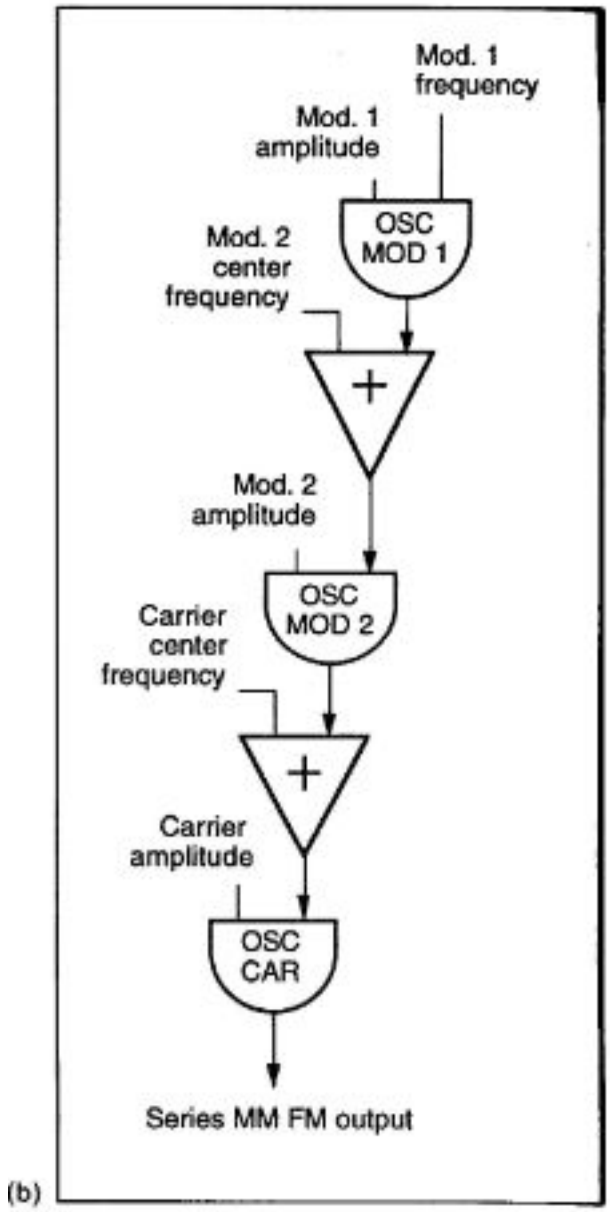
- Example: fmLinearSidebands.pd

15. Extensions

- How can we increase the complexity of the modulator to get more diverse spectra?

16. Extensions: Serial Multiple Modulator FM

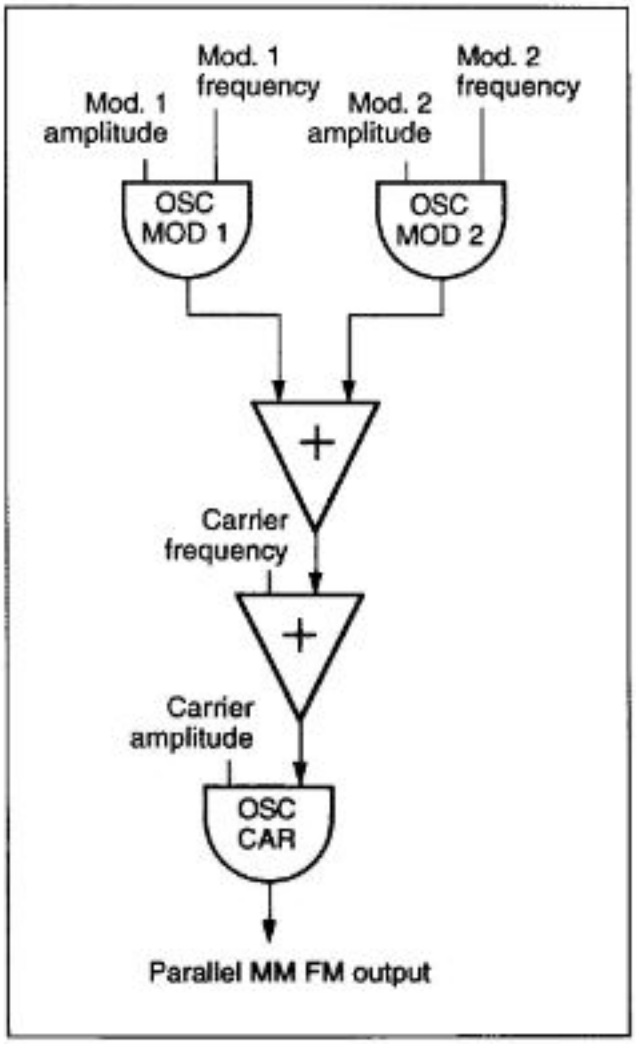
- Modulate the modulator



- Example: fmSerial.pd

17. Extensions: Parallel Multiple Modulator FM

- Combine multiple oscillators in parallel to form a complex modulator



(a)

- Each sideband produced by one modulator is modulated (as if a carrier) by the other modulator (Roads 1996)
- PM implementation (Puckette 2007)
- Example: pmComplex.pd

18. Instrument: Parallel MM PM

- Each voice contains six modulators with preset-provided ratios
- Expressive control over independent index in two groups
- Mappings:

Y1: amplitude

X1: LPF cutoff frequency

Y2: Index of odd modulators between min and max

X2: Index of even modulators between min and max

Buttons: Pitches

- Example: synthPmComplex6.test.pd

19. Chowning and Digital FM

- Parametric order
- Chowning, J. 1973. "The Synthesis of Complex Audio Spectra by Means of Frequency Modulation." *Journal of the Audio Engineering Society* 21(7): pp. 526-534.
- "... a new application of the well-known process of frequency modulation is shown to result in a *surprising control* of audio spectra" (Chowning 1973)
- "the fact that the temporal evolution of the frequency components of the spectrum can be easily controlled is perhaps the most striking attribute of the techniques, for dynamic spectra are achieved only with considerable difficulty using current techniques of synthesis" (Chowning 1973)
- Patented techniques in 1977 (No. 4,018,121)

  Note the inclusion of an amplitude envelope for the FM modulator

United States Patent [19]

Chowning

[11] **4,018,121**

[45] **Apr. 19, 1977**

[54] **METHOD OF SYNTHESIZING A MUSICAL SOUND**

[75] Inventor: **John M. Chowning,** Palo Alto, Calif.

[73] Assignee: **The Board of Trustees of Leland Stanford Junior University,** Stanford, Calif.

[22] Filed: **May 2, 1975**

[21] Appl. No.: **573,933**

Related U.S. Application Data

[63] Continuation-in-part of Ser. No. 454,790, March 26, 1974, abandoned.

[52] **U.S. Cl.** **84/1.01;** 84/101

[51] **Int. Cl.²** **G10H 1/00;** G10H 5/00

[58] **Field of Search** 84/1.01, 1.24, 1.25

[56] **References Cited**

UNITED STATES PATENTS

3,794,748 2/1974 Deutsch 84/1.24

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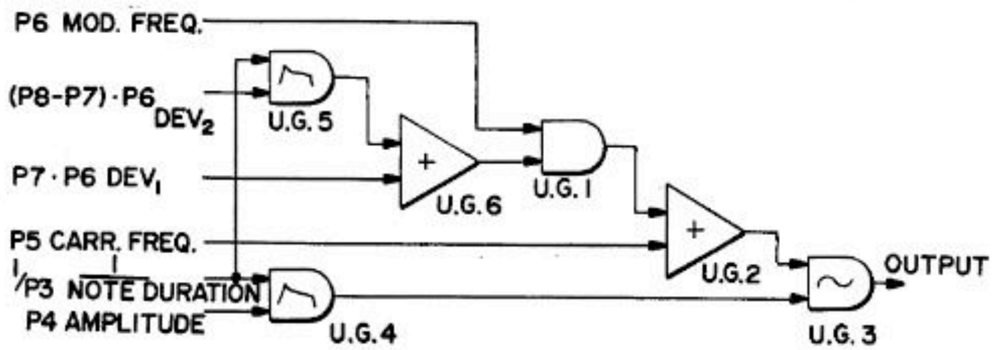
Alan Douglas, "Electrical Synthesis of Musical Tones", Electronic Engineering, July 1953, p. 278.
Alley & Atwood, Electronic Engineering, Second Edition, John Wiley & Sons, Inc., copyright 1966, pp. 564–572.

Primary Examiner—Stanley J. Witkowski
Attorney, Agent, or Firm—Flehr, Hohbach, Test, Albritton & Herbert

[57] **ABSTRACT**

Musical sounds are synthesized by means of frequency modulation with the carrier and modulating frequencies being in the audio range and the modulating index being related to a function to control the bandwidth and evolution in time of the partial frequencies of synthesized sound.

14 Claims, 18 Drawing Figures



20. Chowning: Modulation Index

- Parameter control: derive modulator depth from a scalar of the modulator frequency

  Depth (frequency change) is then always proportional to the modulator frequency

- Given modulating frequency (*m*) and depth (*d*)

  Define a modulation index where $I = d/m$

  $d = I * m$

- Example: fmModIndex.pd

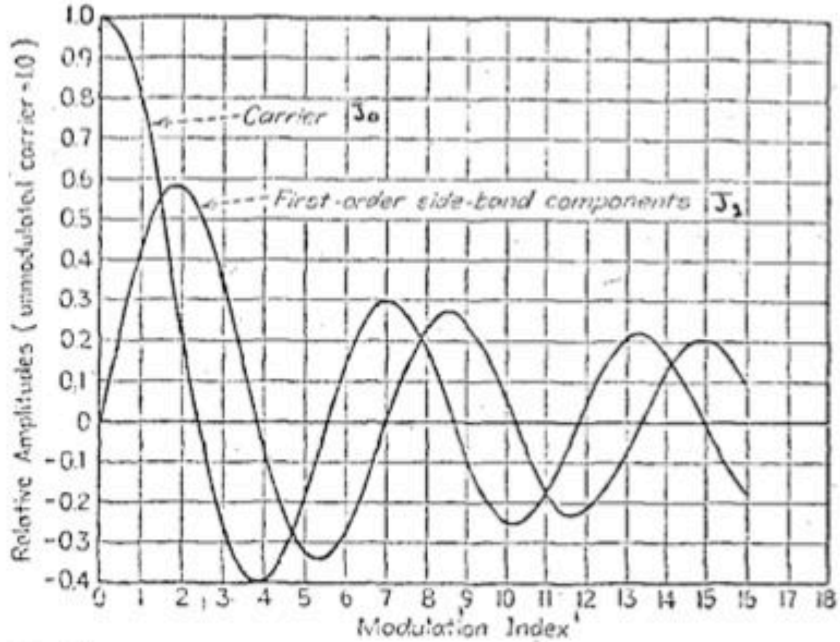
21. PM Modulation Index

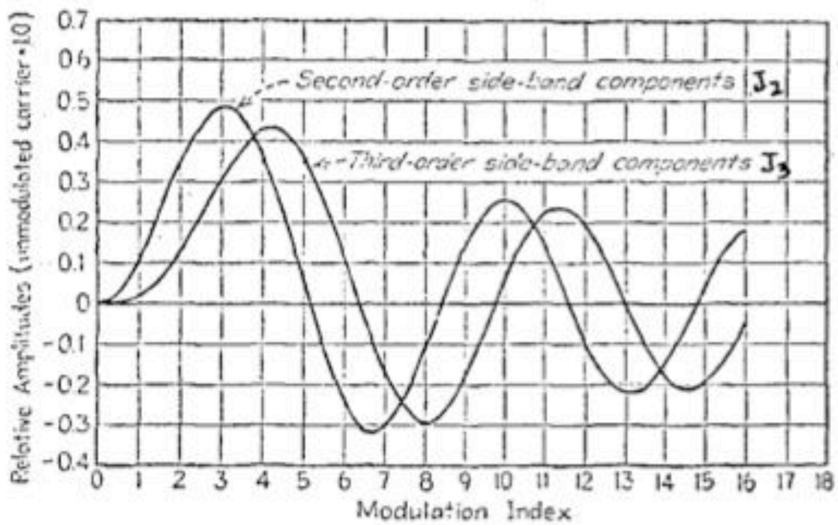
- In PM, modulation-depth is always proportional to a full cycle of the waveform (at any frequency)
- Can convert FM index to PM index by finding the percentage of a complete cycle (divide by 2π)

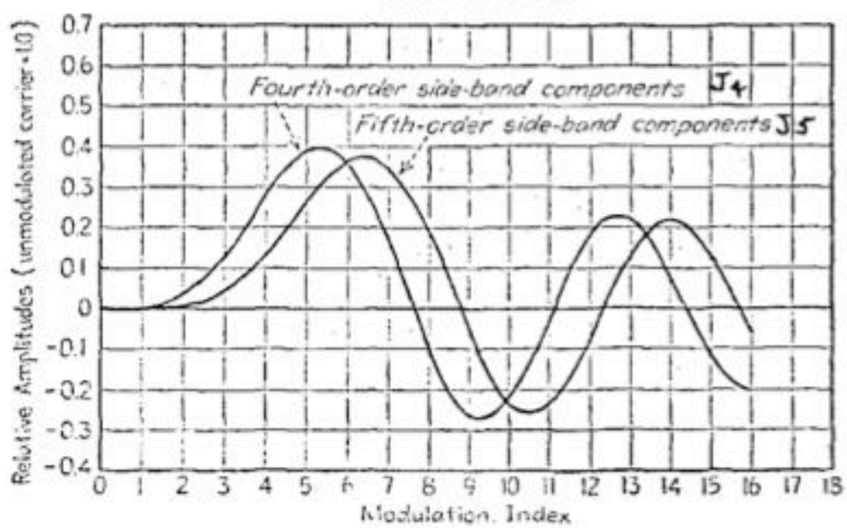
  Example: modIndexCompare.pd

22. Chowning: Amplitude Change in Sidebands as Index Increases

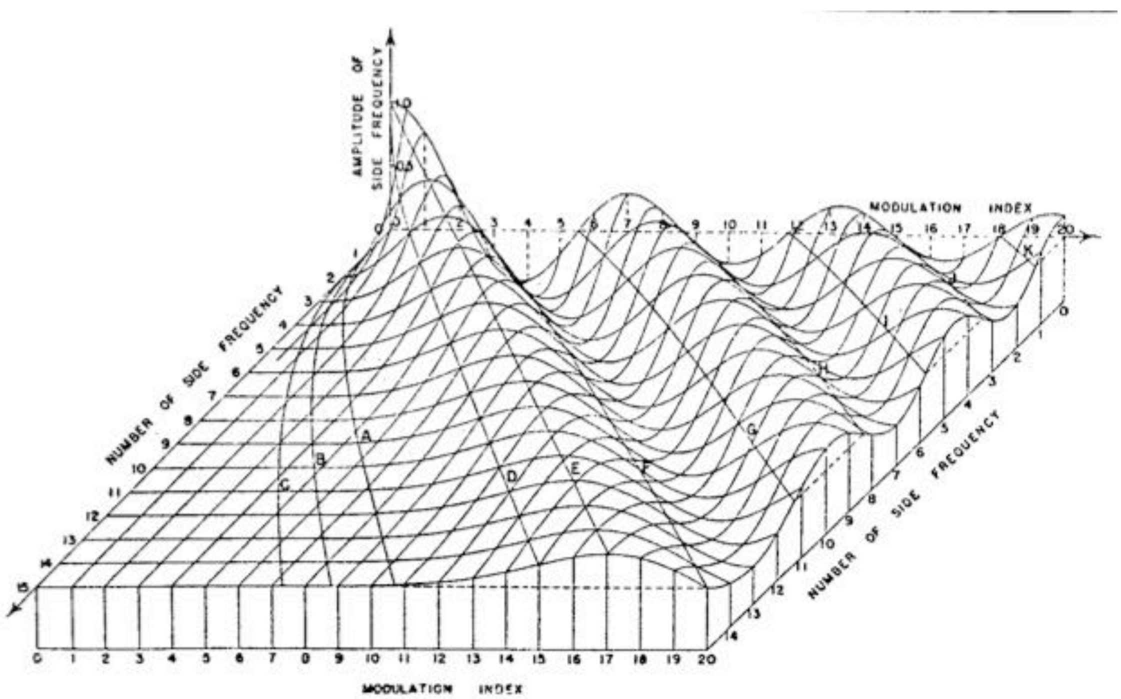
- As the modulation index increases, already-introduced sideband amplitude may increase and decrease
- The pattern of increasing and decreasing, for a single sideband, is determined by the Bessel functions of the first kind and *n*th order
- Given *I* and the sideband number, can get amplitude
- Graph of sidebands 0 through 5 (Chowning 1973)


Carrier J₀
First-order side-band components J₁
Relative Amplitudes (unmodulated carrier = 1.0)
Modulation Index

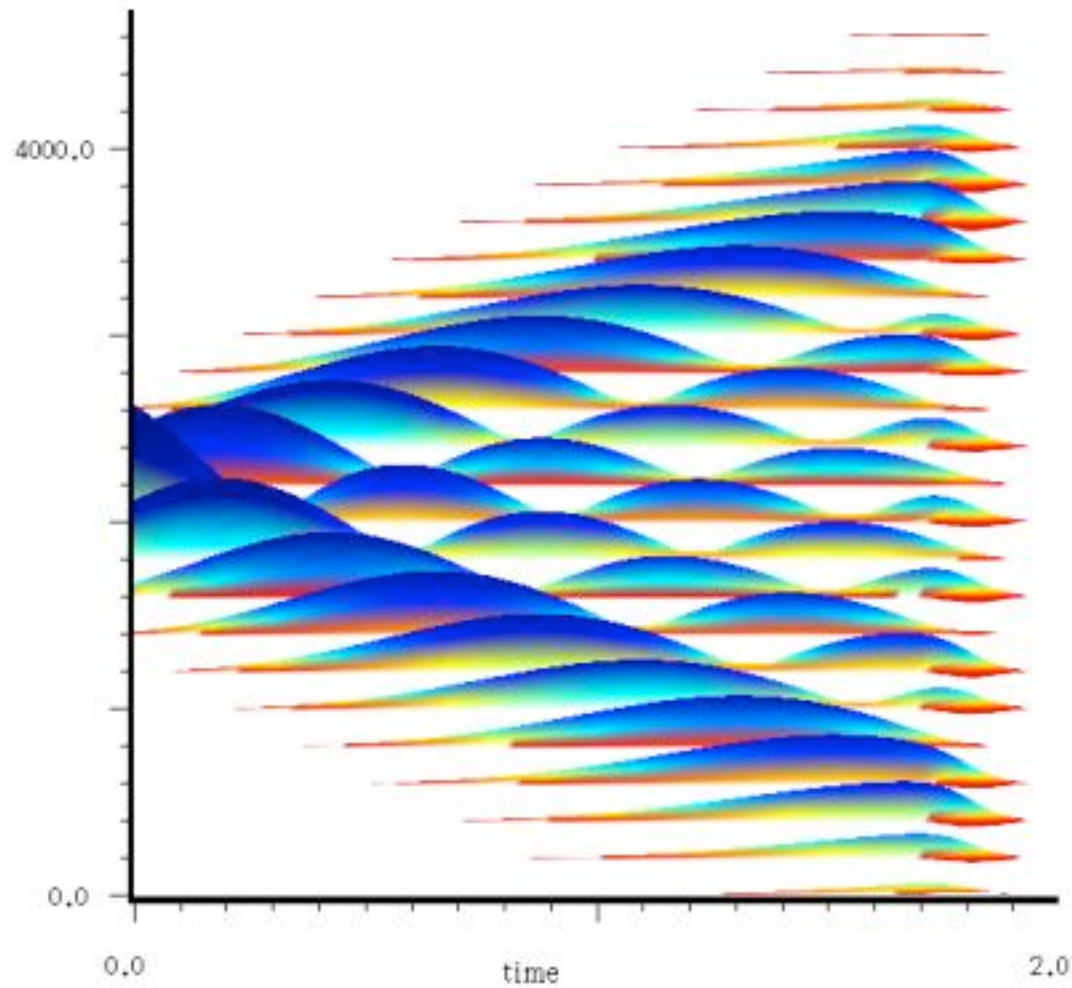

Second-order side-band components J₂
Third-order side-band components J₃
Relative Amplitudes (unmodulated carrier = 1.0)
Modulation Index


Fourth-order side-band components J₄
Fifth-order side-band components J₅
Relative Amplitudes (unmodulated carrier = 1.0)
Modulation Index

- Terrain of Bessels functions for many sidebands across values of I (Chowning 1973)

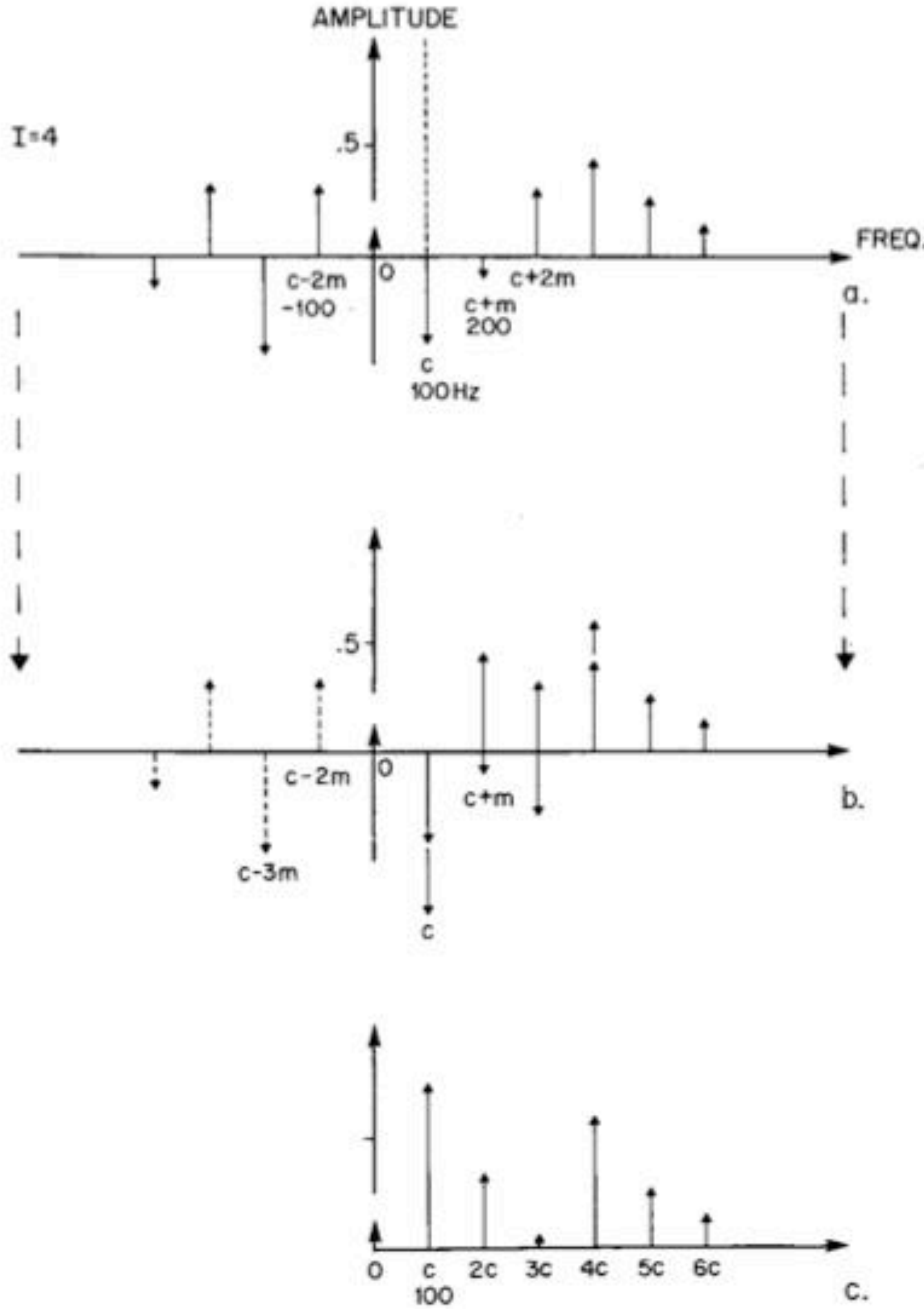


- Alternative view from sweeping FM index from 0 to 10 (C:M is 10/1; Schottstaedt n.d.)



23. Chowning: Sideband Phase

- Bessel functions do not tell us everthing
- As frequencies go below zero, they re-enter the audible range with reversed phase
- Similar to aliasing reflections can happen when we go above the Nyquist frequency
- Given c = 100 Hz, m = 100 Hz, I = 4



24. Chowning: Carrier and Modulator Ratio

- Parameter reduction: coordinate carrier and modulator
- c/m as $N1/N2$

Where $N1/N2$ is a rational number, get harmonic spectra

Sidebands become integer multiples of a fundamental, i.e., harmonic (Roads 1996)

$C = 800$ Hz	(carrier)
$C + M = 1000$ Hz	(sum)
$C + (2 \times M) = 1200$ Hz	(sum)
$C + (3 \times M) = 1400$ Hz, etc.	(sum)
$C - M = 600$ Hz	(difference)
$C - (2 \times M) = 400$ Hz	(difference)
$C - (3 \times M) = 200$ Hz, etc.	(difference)

- Carrier is $N1$th harmonic: if c/m of 4/1, carrier is 4th harmonic
- Example: fmCMRatio.pd
- If *N2* = 1, spectrum contains all harmonics: 1/1, 2/1
- If *N2* is even, get odd harmonics: 1/2, 1/4, 3/2, 3/4, 5/2
- If *N2* is 3, every third harmonic is missing: 1/3, 2/3, 4/3, 5/3

25. Chowning: Inharmonic Spectra

- Why generate harmonic spectra?
- Ratios of irrational numbers result in inharmonic spectra

1 / square root (2) == 1 / 1.41421356

π / square root (3) == 3.14159 / 1.73205081

1 / *e* == 1 / 2.71828

- Example: fmCMRatio.pd

26. Chowning: Dynamic Spectra

- Modulate the modulator: vary *I* over the duration of the event with an envelope
- Can hear the smooth change in amplitude of each sideband, in accordance with Bessel functions
- Common application to increase component bandwidth in attack of sound
- Chowning defined parameters for brass-like tones, woodwind-like tones, and bell, drum, and wood-drum sounds
- Example: fmEnvelopedIndex.pd

27. Instrument: CM Ratio

- Each voice contains a fixed CM ratio and unit-interval spaced index of modulation
- Have manual control over envelopes, with expressive control over high frequencies
- Mappings:

  Y1: amplitude

  X2: LPF cutoff frequency

  Y2: *I* between set min and max

  Buttons: Pitches

- Example: synthFmCmRatio.test.pd

28. Music Example: Arras (1980)

- Composition by Barry Truax
- Truax: Arras (1980)

  Audio (/Volumes/xdisc/_sync/_x/research/_developing/ucsdClass/audio/02Arras.mp3)

29. Music Example: Arras (1980)

- Arras is a continuously evolving texture based on a fusion of harmonic and inharmonic spectra. ... Each harmonic timbre is paired with an inharmonic one with which it shares specific harmonics, and with which it overlaps after each twenty-five second interval ... (CD Liner notes)

- Matched frequencies between 1:2 and 3:8 (Truax 1989)

Table 1. Partials

Ratio	*Spectral Frequencies*																
1:2	1.0		3.0			5.0		7.0	9.0		11.0			13.0		15.0	17.0
3:8	1.0	1.67		3.67	4.33		6.33	7.0	9.0	9.67		11.67	12.3		14.3	15.0	17.0

- Ratios used in the piece (Truax 1989)

Table 2. The harmonic ratios used in *Arras* paired with the inharmonic ones with which they share harmonics

Harmonic Ratio	*Inharmonic Ratio*	f_2	f_3	f_4	f_5
1:2	3:8	7	15	9	17
1:3	4:9	8	17	10	19
1:4	3:8	9	17	7	15
1:5	2:5	9	16	6	14
1:6	—	7	13	7	13
1:7	2:7	6	13	8	15
1:8	3:8	7	15	9	17
1:9	2:9	8	17	10	19

Note: f_2 and f_3 are used as fundamentals for additional components of the background spectrum; f_4 and f_5 are used as carrier frequencies for additional foreground events.

- Truax: Arras (1980)

  Audio (/Volumes/xdisc/_sync/_x/research/_developing/ucsdClass/audio/02Arras.mp3)

30. Familiar FM Sounds

- Commercial digital synths in the 1980s quickly adopted these techniques

  Audio (/Volumes/xdisc/_sync/_x/research/_developing/ucsdClass/audio/05Beat.mp3)

31. Synclavier

- 1978: Synclavier Digital Synthesizer released by New England Digital Corporation
- 1979: Synclavier II employed FM and additive synthesis

- FM Gong patch used in Michael Jackson, “Beat It” (1982)

  Audio (/Volumes/xdisc/_sync/_x/research/_developing/ucsdClass/audio/05Beat.mp3)

32. Yamaha

- 1975: Yamaha obtains a license for Chowning’s FM from Stanford
- 1980: Releases GS1 at $16,000 (Roads 1996)
- 1983: Releases DX7 at $2,000 (Roads 1996)

33. Yamaha: DX7

- Used in countless recordings in the 1980s and later
- HARMONICA 1

  Audio (/Volumes/xdisc/_sync/_x/research/_developing/ucsdClass/audio/dx7HARMONICA1.mp3)

- Many simply used the presets (2:43)

  Audio (/Volumes/xdisc/_sync/_x/research/_developing/ucsdClass/audio/15GotToDo.mp3)

- BRASS 2

  Audio (/Volumes/xdisc/_sync/_x/research/_developing/ucsdClass/audio/dx7BRASS2.mp3)

- EPIANO 1

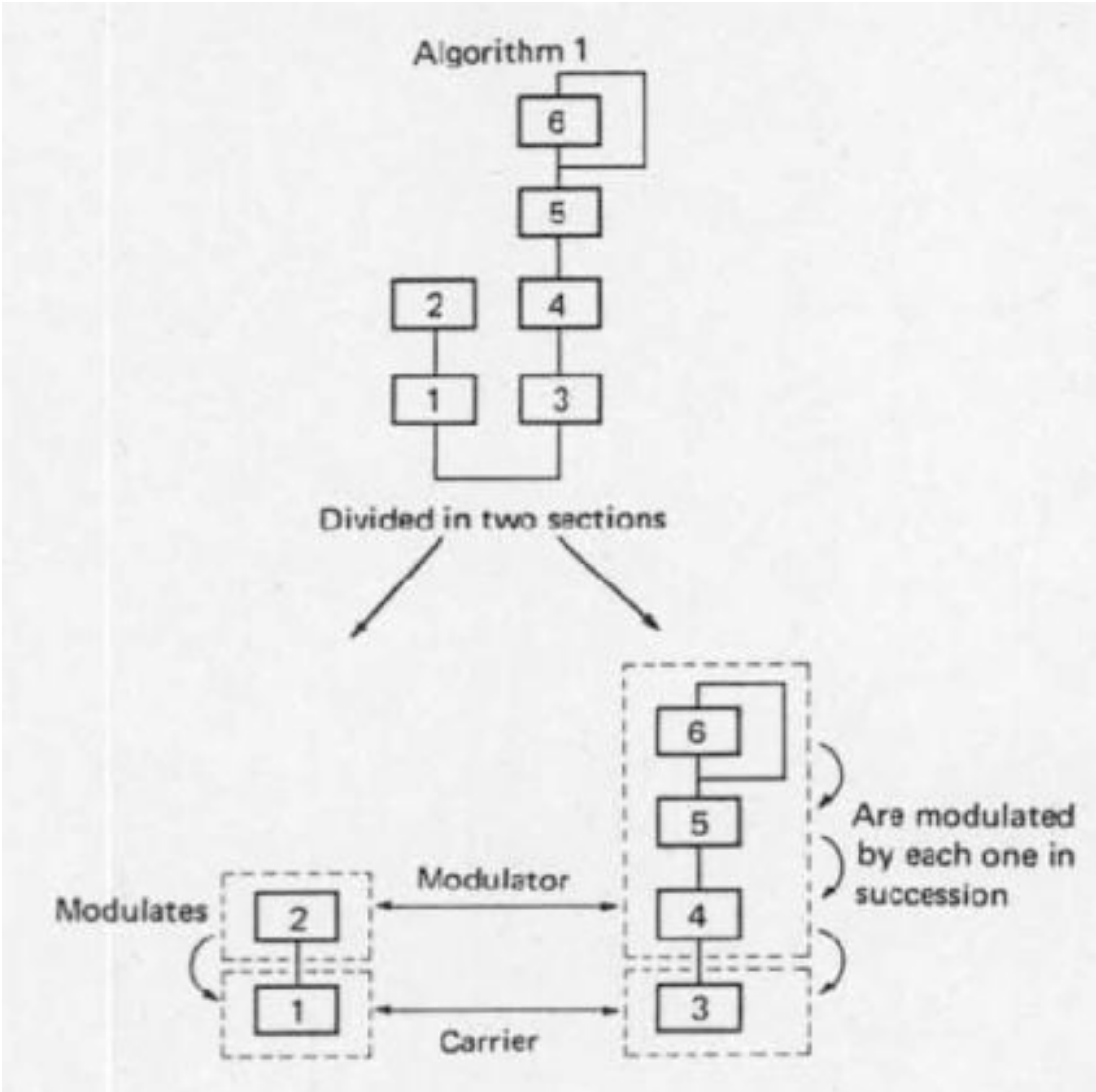
  Audio (/Volumes/xdisc/_sync/_x/research/_developing/ucsdClass/audio/dx7EPIANO1.mp3)

- STRINGS 1

Audio
(/Volumes/xdisc/_sync/_x/research/_developing/ucsdClass/audio/dx7STRINGS1.mp3)

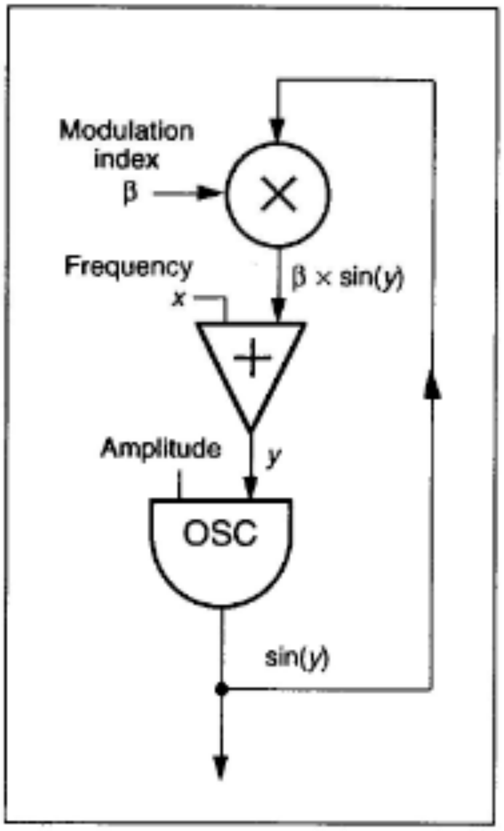
34. Feedback FM

- Yamaha innovated in developing numerous approaches to feedback FM

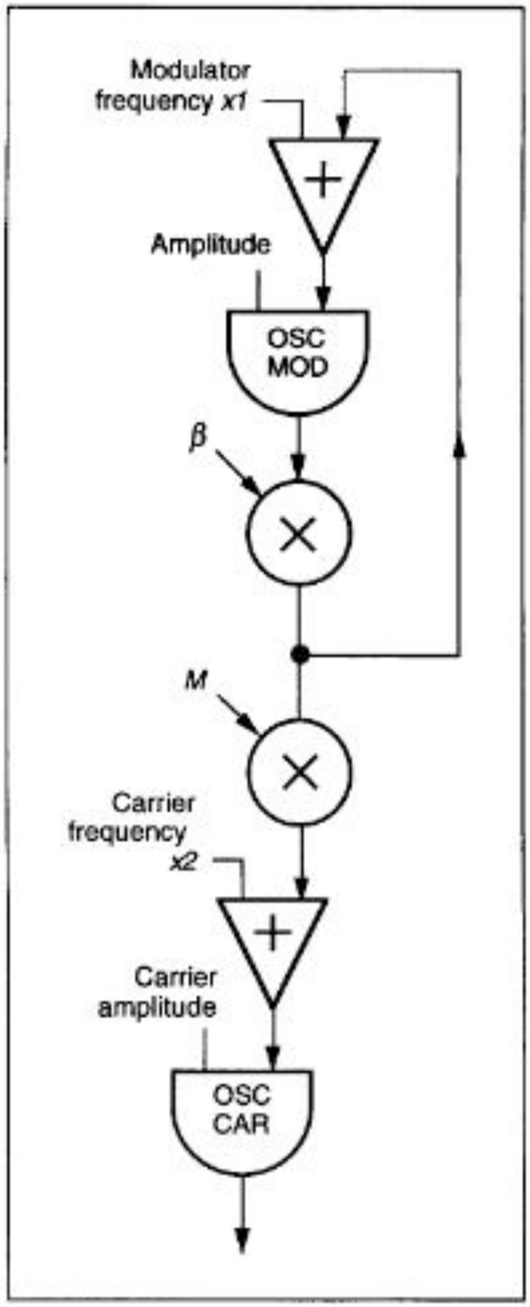


35. Extensions: Feedback FM

- Feedback FM enables a more linear increase in amplitude of sidebands as *I* increases (Roads 1996)
- One oscillator: output of carrier as the modulator (Roads 1996)



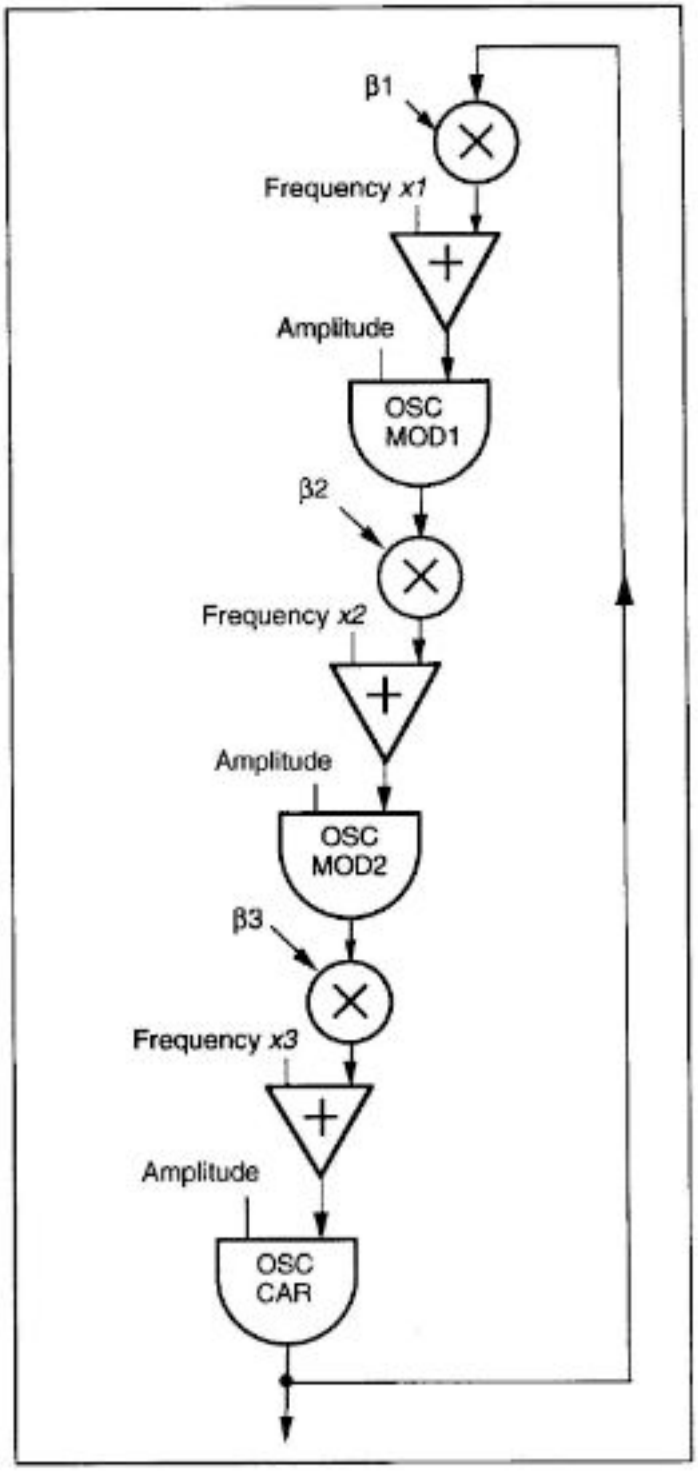
- Two oscillator: output of carrier as the modulator (Roads 1996)



- Three oscillator: output of final oscillator as the modulator of chain of serial modulations (Roads 1996)

  Example: fmFeedbackThree.pd

• Combine exponential FM with a subtractive synth



36. Instrument: Exponential FM Sequencer

- Combine exponential FM with a subtractive synth

- Each voice made of three detuned oscillators frequency-modulated and controlled with an ADSR
- Instrument: synthFmExponential.pd
- Mappings:

  Y1: amplitude

  X2: LPF cutoff frequency

  Y2: LPF resonance

  Buttons: two groups of pitch/amplitude sequenced patterns

- Music Example: Work III

  Audio (/Volumes/xdisc/_sync/_x/research/_developing/ucsdClass/audio/workIIIFmExcerpt.mp3)

37. Conclusions

- FM is not a finished project
- Parameter reduction and organization is critical to getting control over synthesis techniques
- There are always ways to extend modulation: combine in parallel, series, or with feedback other modulators or envelopes

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