Conventions:

I'll use capitals for complex numbers, lower-case for real numbers, and greek letters for anything in radians. (Until I start forgetting. Note the first exception below where R is the sample rate).

Formulas:

A sampled real-valued sinusoid looks like:

$$x[n] = a \cdot \cos\left(\omega n + \phi\right)$$

Here a is the *amplitude*, ω the *angular frequency*, and ϕ the *initial phase*. The angular frequency is in radians per sample, and the initial phase in radians. The frequency can be expressed in cycles per second as:

$$f = \frac{\omega R}{2\pi}$$

A sampled complex-valued sinusoid looks like:

$$X[n] = A \cdot e^{i\omega n}$$

Or rewritten more simply,

$$X[n] = AZ^n$$

where

$$Z = e^{i\omega n} = \cos(\omega n) + i\sin(\omega n)$$

See the Book, section 1.1 (real sinusoids) and then section 7.1 (Time Shifts) for the complex version.

Exercises

- 1. A complex sinusoid X[n] has frequency 11025 Cycles per second has amplitude 50 and initial phase 135 degrees. Another one, Y[n], has the same frequency, but amplitude 20 and initial phase 45 degrees. What are the amplitude and initial phase of the sum of X and Y?
- 2. What are the frequency, initial phase, and amplitude of the signal obtained when X[n] (above) is delayed 4 samples?
- 3. What are the frequency, initial phase, and amplitude of the signal product X[n]Y[n]?

Sonic challenge

Try to synthesize something like http://man104nfs.ucsd.edu/mpuckett/270a.05w/01/crash-stereo.wav.